Foundations of Statistics I: Decision Theory
Problem Set 3

Problem 1 *Prove that any unique Bayes rule is admissible.*

Problem 2 *10 points. Prove that if a minimal complete class $\mathcal{D}$ exists, then it is exactly the class of all admissible estimators.*

Problem 3 *10 points. Let $x_1, \ldots, x_n$ be a random sample from a discrete random variable with distribution $f(x = i|\theta_1, ..., \theta_4) = \theta_i$, for $i = 1, \ldots, 4$. Let θ be the expected value of x given $\theta_1, ..., \theta_4$. Assume loss $L(\theta, a) = (\theta - a)^2$. Use the notation p_i for the number of observed x's that are equal to i.*

1. *Find the maximum likelihood estimator of θ;*
2. *Write an R function to compute the Bayes estimator assuming the prior on $\theta_1, ..., \theta_4$ is a dirichlet distribution with parameters $(\alpha_1, \ldots, \alpha_4)$.*
3. *Take $n = 5$ and fix $\theta_3 = \theta_4 = .1$. Write R code to compare the risk functions of the two estimators as a function of θ_1.*