Foundations of Statistics I: Decision Theory
Problem Set 4

Problem 1 *10 points. Consider the James-Stein estimator, and write a program to graph $R(\theta, \delta_1)/p$ as a function of $\frac{1}{p}\sum \theta_i^2$. Choose your p wisely.*

Problem 2 *[20 points]*

Consider a sample

$$x = (x_1, x_2, \ldots, x_5)$$

where each $x_i \sim Pois(\lambda_i)$. Set $x_2 = x_3 = 0$ and fix x_4 and x_5 to your favorite positive integers. In everything that follows, x_1 will be allowed to vary and $x_2, \ldots, x_5$ will be fixed. You want to estimate λ_1 under squared error loss.

Let δ^1 be the posterior mean of λ_1 assuming that $\lambda_i \sim Exp(1)$, $i = 1, \ldots, 5$

Let δ^{EB} be the empirical Bayes estimate of the posterior mean of λ_1, assuming that $\lambda_i \sim (1-\alpha)I_0 + \alpha Exp(\gamma)$, $\alpha \in (0,1)$, where $\gamma > 0$ is the mean of the exponential.

Write computer programs to perform the following tasks:

1. *Using an empirical Bayes approach, determine maximum likelihood estimates of α and γ. These will be functions of x.*
2. *graph δ^{EB} and δ^1 versus x_1; determine whether the relationship is linear or nonlinear; choose your resolution and range so that your conclusion is supported by the figure;*
3. *graph the risk functions of δ^{EB} and δ^1, as you vary λ_1 and fix the other coordinates $\lambda_i = x_i$ $i \geq 2$. These risk functions are averages over all possible values of x_1.*

Problem 3 *x is exponential with rate θ. Consider the class of estimators $\delta_c(x) = cx$ and determine the best estimator of $1/\theta$ under squared error loss. Show that the estimator is a genearlized Bayes esitimator and discuss its admissibility.*